Annual and Final Report: September 1, 1997 to August 31, 1998
Grant Number F49620-96-1-0397

Constitutive Relations in Granular Soils

James T. Jenkins
Department of Theoretical and Applied Mechanics
Cornell University, Ithaca, NY, U. S. A.

M. A. Curt Koenders
School of Applied Physics
Kingston University, Kingston upon Thames, Surrey, U. K.

In the past year, we continued our efforts on the development of two models for the mechanical response of a random aggregate of identical spheres that interact through frictional elastic contacts.

In the first model, we regard the granular material as a collection of two-particle systems embedded in an average medium and use the balance of force and moment to solve for the motion of the pair. This model provides the simplest context in which to introduce fluctuations in particle displacements and spins. The equations of force and moment balance are sufficient for their determination. Because of its simplicity, the applicability of the model is limited to small or moderate stress ratios.

In the second model, we focus on a small assembly consisting of one particle surrounded by others and determine the motion of a coupled system of these small assemblies. Here the displacements and spins of as many as twelve particles are determined by satisfying force and moment in a least squares fashion. The complexity of this model permits its application to situations involving moderate to large stress ratios. As the stress ratio increases, coefficient matrices become singular, indicating the need for more elaborate descriptions, and foreshadowing the development of shear bands.

In the context of each model it is possible to predict the average incremental stiffness for a given heterogeneous assembly. It remains to develop the means to predict the evolution of the appropriate measures of the geometry and contact stiffness that take place as a strain path is traversed. It should be said that the latter has been studied in the context of a particularly simple continuum description, and it has been demonstrated that structures formation can take place. This effect is rather a novel element in the modeling of granular materials. It also has

REPORT DOCUMENTATION PAGE

AFRL-SR-BL-TR-98-

0833

Public reporting burden for this collection of information is estimated to average 1 hour per response, incluc ... urces, gathering and maintaining the data needed, and completing and reviewing the collection of information. Se ... of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Servi ... ferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Pap

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REP
	August 1998	Final Technical Report 1 Aug 96 to 30 Sep 98

4. TITLE AND SUBTITLE
Constitutive Relations in Granular Soils

5. FUNDING NUMBERS
F49620-96-1-0397
2302/CS

6. AUTHOR(S)
James T. Jenkins

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Department of Theoretical and Applied Mechanics
Cornell University
Ithaca, NY USA

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NA
801 N. Randolph Street Room 732
Arlington VA 22203-1977

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
F49620-96-1-0397

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

In the past year, we continued our efforts on the development of two models for the mechanical response of a random aggregate of identical spheres that interact through frictional elastic contacts. In the first model, we regard the granular material as a collection of two-particle systems embedded in an average medium and use the balance of force and moment to solve for the motion of the pair. This model provides the simplest context in which to introduce fluctuations in particle displacements and spins. The equations of force and moment balance are sufficient for their determination. Because of its simplicity, the applicability of the model is limited to small or moderate stress ratios. In the second model, we focus on a small assembly consisting of one particle surrounded by others and determine the motion of a coupled system of these small assemblies. Here the displacements and spins of as many as twelve particles are determined by satisfying force and moment in a least squares fashion. The complexity of this model permits its application to situations involving moderate to large stress ratios. As the stress ratio increases, coefficient matrices become singular, indicating the need for more elaborate descriptions, and foreshadowing the development of shear bands.

14. SUBJECT TERMS

15. NUMBER OF PAGES
3

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

applications in other fields of non-linear/evolutionary continuum mechanics, such as non-Newtonian seepage flow.

Two papers containing the results of the research on the two models are currently being prepared. In the course of the study, we published the following papers (not including those presented at Landsdowne), organized an international meeting, and edited a book:

1. J.T. Jenkins and M.A. Koenders. Inelastic behavior of random arrays of identical disks. In Powders and Grains '97 (R.B. Behringer and J.T. Jenkins, Eds.) pp 239-242, Balkema: Rotterdam, 1997.

We outline a program for obtaining the incremental stress-strain relations for a heterogeneous medium from a study of small local assemblies. As an example of such an assembly we consider a pair of disks that interact with each other and with their other neighbors through elasto-frictional contacts. We indicate how a solution for the increments in the translations of the centers of the two disks and the rotations about their centers can be obtained from the consideration of force and moment equilibrium when the neighbors of the pair are constrained to translate and rotate with the average motion.

2. M.A. Koenders, Modeling techniques for granular media using small assemblies. Powders and Grains '97 (R.B. Behringer and J.T. Jenkins, Eds.) pp 353-356, Balkema Rotterdam, 1997.

A granular medium is viewed as a heterogeneous continuum. Various homogenization techniques are available. These are briefly discussed and then, using the theory of randomly heterogeneous materials, it is shown that packing inhomogeneity is a non-trivial effect for granular media with frictional interaction at high stress ratio. To demonstrate this a primitive stiffness for a small assembly is used. It is shown that for high stress ratio packing heterogeneity - together with other effects - is not a negligible effect.

3. M.A. Koenders. A model of a granular assembly as a structured material. In Mechanics of Deformation and Flow of Particulate Materials" (A. Misra, C.S. Chang, R.Y Liang and M. Babic, Eds.) pp 11-23, ASCE: New York, 1997.

The mechanical behavior of a two dimensional granular medium is simulated by mapping it onto a model material. Inspired by numerical simulations in which an assembly has been studied a double shearing geometry is chosen. The blocks of this model interact with neighbors via

contacts and the blocks have rough edges to model dilatancy. The model is made valid for lower stress ratios than the traditional double shearing models by letting a fraction of the contacts slip. The fraction depends on the stress ratio of the assembly as a whole. A biaxial cell test is simulated based on these concepts and features of the stress strain curves are recovered.

4. M.A. Koenders. The evolution of spatially structured elastic materials using a harmonic density function. Phys Rev E, 56 5585-5593 (1997).

A study is presented of the evolution of a heterogeneous material that is subjected to an increment-by-increment, deviatoric strain path. The internal evolution of the stiffness components may be effected in various ways; some simple ones are studied here and the resulting structure formation is detected via the spatial Fourier transform of the stiffness fluctuations.

5. J.T. Jenkins and R.B. Behringer (Editors) Powders and Grains 97, 523 pp. Balkema Rotterdam, 1997.

The proceedings of a meeting held in Durham, NC involving 120 engineers and physicists devoted to the study of particulate materials.

6. M.A. Koenders (1998). Effects of microstructure and non-linearity in heterogeneous materials. Journal of Physics D. 31, 15, 1875-1882.

The estimate of overall moduli is studied using a perturbative technique developed by Kroner and applied to a permeability problem. Correction terms to allow for the discrete character of a granular medium and the non-linearity of the flow are introduced. The combination of heterogeneity and non-linearity is shown to lead to the formation of direction dependent features. It is shown how these can be predicted by theoretical means and an experiment is also described in which these features have been observed.

7. M.A. Koenders. Effects of evolution on a heterogeneous material. Proc. 12th Engineering Mechanics Conference (eds H. Murakama and J.E. Luco). May 12-20. San Diego. USA (1998), pp 1701-1704.

The evolution of a heterogeneous material is studied. Two evolution rules are applied, one based on the isotropic strain, the other on the deviatoric strain. Especially in the latter case, interesting patterns form.